SIGNAL FILTERING USING ORTHOGONAL POLYNOMIALS AND REMOVAL OF EDGE EFFECTS

FIELD OF THE INVENTION

The present invention relates to a method and device for filtering an input signal containing wanted signal components and unwanted signal components.

BACKGROUND OF THE INVENTION

Characteristics of typical signals, disturbances, noise and interference including myoelectric (EMG) signals, cardiac (ECG) signals, movement-induced disturbances, background noise, and interference from the mains will be first reviewed.

Myoelectric (EMG) signal

An EMG signal reflects the nervous activation of a muscle or a group of muscles, and presents the character of a band-limited noise in extra-muscular readings. The strength and spectral contents of an EMG signal are determined by the following parameters:

(1) the number of motor units (muscles) recruited;
(2) the repetition rate of motor unit activation;
(3) the distance between the source (fibers of the muscle(s)) and the electrode(s) used to sense the EMG signal;
(4) the width of the innervation zone;
(5) the propagation velocity of the activation potentials;
(6) the electrode configuration including, for example, the distance between electrode plates, the parallel or perpendicular alignment of the electrode plates with respect to the direction of the fibers of the muscle(s), etc.; and
(7) other parameters such as the length of the fibers of the muscle(s), the position of

the electrode(s) with respect to both the innervation zone and the ends of the fibers of the muscle(s), etc.

For many muscles, except very short muscles, the above-mentioned parameters produce a spectrum whose amplitude increases from zero at zero frequency (Direct Current (DC)), presents a peak in the region of 40-100 Hz and, then, decreases gradually as the frequency increases.

Models that take into consideration the above listed parameters show that these parameters are multiplicative in the frequency domain, indicating that the time dependent signals are multiple convolutions of the phenomena associated with these parameters. The time representation of an EMG signal is thus very complex, and most attempts to process such a signal has consequently involved filtering of the EMG signal through certain frequency-filtering characteristics.

Electrocardiogram (ECG) signal

An ECG signal is generated from electrical activity of the heart, which is rather coherent in space and time. Thus, an ECG signal has a deterministic character in the time domain except under certain pathological conditions such as fibrillation. In the frequency domain, maximum density of the spectral energy is found in the region around 10-15 Hz with the spectral energy density decreasing as the frequency increases.

Since contributions to the ECG signal are synchronized, the ECG signal has a much higher energy than an EMG signal. Moreover, significant ECG contributions are located in the frequency bandwidth where the density of energy of an EMG signal is maximal, whereby ECG is likely to disturb an EMG signal.

Movement induced disturbance

Movement of the electrodes cause variations in the electrode contact resistance and electrode contact capacitance, which modulate the (half-battery) voltage across the

metal-tissue junction to cause a corresponding disturbance. This disturbance, often referred to as "varying baseline", has high-amplitude low-frequency spectral components since it is caused by a slow mechanical movement. The frequency characteristic of this disturbance is often lower in frequency than the frequency characteristic associated with the time window of the signal epoch and, thus, this disturbance manifests itself as a large offset DC component.

It is also observed that variations in the baseline are equivalent to variations in the spectral components above DC (zero frequency). Therefore, the large potential behind baseline variations (magnitude of the order of, for example, one volt) also makes the spectrum density of the variation significant.

Background noise

In addition to the above-mentioned signal contributions, there is a noise component. This noise component has a rather unspecific character and originates from a number of different sources. It is often modelled as a random component with constant (or slowly varying) spectral density in the whole frequency bandwidth of interest.

Interference from the mains

Interference from the mains has a well-defined signal shape with a fixed frequency. Unknown parameters of this interference are the phase and the amplitude, which may vary considerably.

Currently used filter techniques to separate signal contributions

In the following description, some of the problems associated with the separation of signal components such as EMG, ECG, baseline variations, and noise will be discussed. Drawbacks associated with currently used signal filtering methods will also be considered.

EMG signal vs. movement induced disturbance

Although rather separated in frequency, the high energy density of the movement induced disturbances becomes significant also in the frequency bandwidth where EMG signals have a maximum amplitude. Recursive filtering is often used to remove movement induced disturbances from EMG signals. However, recursive filtering presents the drawback of suffering from the long time it takes to forget the high energy input from a sudden baseline variation. For that reason, recursive filtering should be avoided.

EMG signal vs. background noise

The spectra of EMG signal and background noise overlap and both have a character of random noise. Optimum (Wiener) filtering can be used, but a low-pass filter cutting off the high frequency portion of the noise can be sufficient since this will remove the major portion of the background noise.

EMG signal vs. ECG signal

As indicated in the foregoing description, EMG and ECG signals have partly overlapping power spectra. Strong ECG signals create significant spectral energy density at frequencies where energy peaks of the EMG signals appear. Efficient filtering to remove an ECG signal from an EMG signal or vice versa should use the deterministic character of the ECG signal in the time domain.

ECG signal vs. movement induced disturbance

ECG signals and movement induced disturbances have respective spectra with a considerable overlap and are difficult to separate in the frequency domain. The deterministic character of ECG in the time domain is often utilized, in particular for averaging QRS complexes. A drawback of such filtering is that it introduces in the output from the filter a delay having a duration as long as ten (10) seconds.

ECG signal vs. background noise

Although ECG signals and background noise overlap in the low frequency region of the spectrum, the ECG signal is mostly dominating and, therefore, a low-pass filtering is normally sufficient to remove most of the background noise from an ECG recording.

Movement induced disturbance vs. background noise

Although this case is of interest in quality control of the signal, this issue will not be further elaborated in the present specification.

Interference from the mains and power line interference

A common technique for removing an interference from the mains and a power line interference is to process the disturbed signal through a notch filter which is quite sharp at the particular frequency of these interferences. This solution presents the drawback that a notch filter introduces large phase shifts in the remaining part of the signal. As mentioned hereinabove, the nature of these interferences is "a priori" well known, and their frequency is mostly constant. Only the start instant within the oscillation, the phase, and the amplitude are unknown and have to be calculated. This "a priori" knowledge implies that single periods or even portions of the interference oscillation can be used to perform the latter calculation.

From the above-described characteristics of the various signal contributions, the following acknowledgments can be made:

- Filtering in the frequency domain requires very steep filters to extract the EMG signal and to suppress the ECG signal and the interference from the mains. Steep filters present the drawback of being associated with large phase shifts and ringing.

- Since the baseline variations are large and slow, recursive filtering should be avoided since these filters have a memory of past events.

- Filtering should occur over finite epoch lengths, and Infinite Impulse Response (IIR) filters should not be used.

- Filters designed from a frequency-domain point of view present the drawbacks of having 1) slow convergence since the trigonometric series are not well suited to describe baseline variations, 2) remaining memory of earlier events due to their (often) recursive character, and 3) large phase shifts and ringing in proportion of their sharpness.

In view of the above, a new filtering technique is needed to efficiently isolate or remove one or many of the above discussed signal contributions from an input signal.

SUMMARY OF THE INVENTION

More specifically, in accordance with the present invention, there is provided a method of filtering an input signal containing wanted signal components and unwanted signal components, comprising modeling the input signal as a set of polynomials, identifying polynomials from the set to model the unwanted signal components, and removing the unwanted signal components from the input signal by removing the polynomials identified as modeling the unwanted signal components from the set of polynomials to thereby leave in the input signal only the wanted signal components.

The present invention also relates to a device for filtering an input signal containing wanted signal components and unwanted signal components, comprising means for modeling the input signal as a set of polynomials, means for identifying polynomials from the set to model the unwanted signal components, and means for removing the unwanted signal components from the input signal, wherein the unwanted signal components removing means comprises means for removing the polynomials identified as modeling the unwanted signal components from the set of polynomials to thereby leave in the input signal only the wanted signal components.

Further in accordance with the present invention, there is provided a method of filtering an input signal containing wanted signal components and unwanted signal components, comprising modeling the input signal as a set of polynomials, identifying polynomials from the set to model the wanted signal components, and outputting the polynomials identified as modeling the wanted signal components as an estimate of the input signal substantially free from the unwanted signal components.

The present invention further relates to a device for filtering an input signal containing wanted signal components and unwanted signal components, comprising means for modeling the input signal as a set of polynomials, means for identifying polynomials from the set to model the wanted signal components, and means for outputting the polynomials identified as modeling the wanted signal components as an estimate of the input signal substantially free from the unwanted signal components.

Still further in accordance with the present invention, there is provided a method of filtering an input signal containing wanted signal components and unwanted signal components, comprising modeling the input signal as a set of polynomials, determining, for each polynomial, a weighting coefficient indicative of signal strength, and summing the weighting coefficients to provide an estimate of the strength of the wanted signal components.

The present invention still further relates to a device for filtering an input signal containing wanted signal components and unwanted signal components, comprising means for modeling the input signal as a set of polynomials, means for determining, for each polynomial, a weighting coefficient indicative of signal strength, and means for summing the weighting coefficients to provide an estimate of the strength of the wanted signal components.

The foregoing and other objects, advantages and features of the present invention will become more apparent upon reading of the following non-restrictive description of illustrative embodiments thereof, given by way of example only with reference to the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

In the appended drawings:

Figure 1 is a schematic flow chart illustrating a technique according to a non-restrictive illustrative embodiment of the present invention, using orthogonal polynomials to model an input, contaminated signal;

Figure 2 illustrates, as an example, normalized Legendre orthogonal polynomials in an interval $-1<x<1$, of degrees zero (0) to seven (7) used for filtering finite length signal epochs;

Figure 3 is a schematic flow chart of a first method according to a non-restrictive illustrative embodiment of the present invention, for filtering an input signal having wanted signal components and unwanted signal components;

Figure 4 is a schematic flow chart of a second method according to a non-restrictive illustrative embodiment of the present invention, for filtering an input signal having wanted signal components and unwanted signal components;

Figure 5 is a schematic flow chart of a third method according to a non-restrictive illustrative embodiment of the present invention, for filtering an input signal having wanted signal components and unwanted signal components;

Figure 6 is a schematic flow chart of a technique according to a non-restrictive, illustrative embodiment of the present invention, for eliminating edge effects;

Figure 7 illustrates a non-restrictive example of contaminated input signal (upper trace) splitted into overlapping windows, including odd windows (middle trace) and even windows (lower trace);

Figure 8 is a non-limitative example of unwanted signal components modeled in the

odd and even windows of Figure 7 by signals synthesized from Legendre polynomials of orders zero (0) to seven (7);

Figure 9 are examples of signal parts remaining after removal from the input signal of the signals of Figure 8, modeling the unwanted signal components;

Figure 10 illustrates non-limitative examples of edge effect suppressing functions in the respective odd and even windows of Figure 7;

Figure 11 are examples of weighted signal parts obtained by multiplying the signal parts of Figure 9 with the edge effect suppressing functions in the respective odd and even windows of Figure 7; and

Figure 12 are examples of the input contaminated signal (upper trace) and the filtered signal (lower trace) obtained by summing the weighted signal part of the various overlapping windows.

DETAILED DESCRIPTION OF THE ILLUSTRATIVE EMBODIMENT

From the analysis of the EMG signals, ECG signals, movement induced disturbances, background noise, and interference from the mains made in the "background of the invention" of the present specification, the following conclusions can be drawn:

- The "a priori" knowledge of the input signal could be used for designing the filtering technique. One example is the interference from the mains, which has substantially sinusoidal shape and fixed frequency.

- The filtering technique could model the disturbances, offsets, trends, noise, etc., in a more efficient manner than the Fourier transform is capable of. Other functions requiring a smaller set of terms could be used to more efficiently model the essential features of the signal.

- DC components could be removed from finite length epochs since these components have frequencies lower than the lowest frequency associated to the epoch.

- Trends could be removed from the signal. The reason for this time domain processing is that a ramp has a large number of harmonics when decomposed in the Fourier domain.

- The frequency characteristics of the filtering technique could be preserved at higher frequencies in order to fully benefit from experiences of spectral filtering.

- Since trigonometric functions do not fulfill the above requirements in the low frequency region of the spectrum, polynomials or other functions could be used to comply with the requirements associated to DC, ramp and oscillatory behaviour at higher frequencies.

- Orthogonal sets of functions could be used in the filtering technique so that modifications of the filter components do not mutually influence each other.

Keeping the above in mind, the non-restrictive illustrative embodiments of the method and device according to the present invention are concerned with a filtering technique intended primarily for supplying filtered signals from contaminated signals of various kinds. Although this filtering technique could be used in a plurality of different applications, the illustrative embodiments relate to filtering of bioelectric signals.

More specifically, the description of the illustrative embodiments of the method and device in accordance with the present invention will focus on the filtering of EMG signals (wanted signal components) contaminated with ECG signals, movement induced disturbances, and background noise (unwanted signal components). The goal of the illustrative embodiments is to obtain a filtered EMG signal from a contaminated esophageal recording in view of controlling a mechanical ventilator.

Another possible application in the bioelectric field is to remove baseline variations (unwanted signal components) from ECG signals (wanted signal components) in view of obtaining immediate estimates of ST-depressions instead of averaging these ST-depressions over a number of QRS complexes, which delays the estimation process.

In yet another possible application, wanted signal components comprise internal variations of a QRS complex of an ECG signal and the unwanted signal components comprise a remaining part of the ECG signal. This enables investigation of internal variations of electric propagation within the heart muscle for the purpose of studying heart arrhythmia.

Filtering with orthogonal polynomials

A non-restrictive illustrative embodiment of the present invention proposes to use orthogonal, normalized polynomials to model the unwanted signal components in the contaminated signal. Once modeled, it is easier to remove the unwanted signal components from a contaminated input signal in order to obtain a corresponding, filtered signal substantially free from these unwanted signal components.

Another non-restrictive illustrative embodiment of the present invention is to model the wanted signal components and to use this model as an estimate of the filtered signal generally free from the unwanted signal components.

A non-restrictive illustrative embodiment of a method for modeling signal components of a contaminated signal will now be described.

Operation 11 (Figure 1)

Referring to Figure 1, let's consider a signal epoch *S(t)* in a limited time interval.

Operation 12 (Figure 1)

The time scale of the epoch *S(t)* is normalized. For that purpose, the limited time interval under consideration is shifted and scaled into a new variable *x* in an interval -1<x<1. The signal epoch under consideration then becomes *S(x)*.

Operation 13 (Figure 1)

The signal epoch *S(x)* is described in terms of a set of orthogonal polynomials $Q_n(x)$. A first initial polynomial $Q_0(x)$ has a constant value, for example proportional to the DC component. A second initial polynomial $Q_1(x)$ has a constant slope, for example proportional to a trend. Thus:

$$Q_0(x) = \text{Constant} \qquad (1)$$

and

$$Q_1(x) = \text{Constant} \times x \qquad (2)$$

Another condition requires the polynomials to be orthogonal, i.e. that:

$$\int_{-1}^{+1} Q_m(x)\ Q_n(x)\ f(x)\ dx = \begin{cases} 0 & \text{for } m \neq n \\ K_n \neq 0 & \text{for } m = n \end{cases} \qquad (3)$$

where *f(x)* is a function of *x* and K_n is a constant depending only on the order *n*.

The conditions given in equations (1) to (3) are fulfilled by several types of polynomials, which are orthogonal in the interval –1<x<1. Non-restrictive examples are the Legendre polynomials, Tchebyshev T-polynomials, and Tchebyshev U-polynomials.

As a non limitative example, Figure 2 illustrates polynomials, orthogonal in the interval –1<*x*<1, of degrees zero (0) to seven (7) used for filtering finite length signal epochs. The functions in the plot of Figure 2 are normalized Legendre polynomials.

The following Table 1 lists some of the properties of the above-mentioned Legendre polynomials, Tchebyshev T-polynomials, and Tchebyshev U-polynomials.

Table 1:
Some characteristics of polynomials being orthogonal in the interval $-1<x<1$

Type	$f(x)$	K_n	$Q_0(x)$	$Q_1(x)$	$Q_2(x)$	$Q_3(x)$
Legendre	1	$2/(2n+1)$	1	x	$(3x^2-1)/2$	$(5x^3-3x)/2$
Tchebyshev T	$(1-x^2)^{-½}$	$\pi/2$ (§)	1	x	$2x^2-1$	$4x^3-3x$
Tchebyshev U	$(1-x^2)^{½}$	π	1	$2x$	$4x^2-1$	$8x^3-4x$

(§) $K_0 = \pi$. Also, in the synthesis (equation (4)), a factor of 0.5 is required for n=0.

The contaminated input signal can be synthesized from a sum of these polynomials:

$$S(x)=\sum_{n=0}^{\infty} C_n \; Q_n(x) \qquad (4)$$

where

$$C_n = \frac{1}{K_n} \int_{-1}^{+1} S(x) \; Q_n(x) \; f(x) \; dx \qquad (5)$$

In practice, a limited number of terms is used. One reason is that terms of increasing order reflect increasing frequencies, which corresponds to regions containing little information.

The filtering technique can be conducted in accordance with different methods. The choice of the method will depend on the number of numeric calculations involved.

Obviously, the longer the series of polynomials the more complicated the filtering procedure. For that reason, too long a series of polynomials will be avoided.

First method (Figure 3)

In accordance with a non-restrictive illustrative embodiment of the present invention, a first filtering method comprises the following operations:

1. The particular orders of the polynomials associated with the unwanted signal components are identified (Operation 31);

2. The unwanted signal components are modeled as a sum of polynomials (equation (4) and Operation 32) weighted by means of the coefficients C_n (equation (5)); and

3. This model of the unwanted signal components is removed from the input, contaminated signal, more specifically from the other polynomials of the original set to obtain the filtered signal substantially free from these unwanted signal components (Operation 33).

Second method (Figure 4)

In accordance with a non-restrictive illustrative embodiment of the present invention, a second filtering method comprises the following operations:

1. The particular orders of the polynomials of the set associated with the wanted signal components are identified (Operation 41);

2. The wanted signal components are modeled as a sum of polynomials (equation (4) and Operation 42) weighted by the coefficients C_n; and

3. The sum of weighted polynomials is outputted as an estimate of the filtered signal, substantially free from the unwanted signal components (Operation 43).

Third method (Figure 5)

This third method is used when only a representation of the strength of the signal is wanted. The third method can proceed according to the above-described second method (Figure 4) while avoiding the signal synthesis (modeling of the wanted signal components as a sum of weighted polynomials as described in equation (4)), using instead the coefficients C_n to model the strength of the wanted signal components.

More specifically, according to this third method:

1. The coefficients C_n are used as estimates for the signal strength (Operation 51) to thereby model the strength of the wanted signal components.

2. The set of coefficients C_n are then added to provide an estimate of the strength of the wanted signal components (Operation 52). As a non-restrictive illustrative example, the coefficients C_n are summed on a square law basis to give an estimate of the signal power, for example by summing squared coefficients C_n. This summation on a square law basis is performed with additional weighting for the order n of the polynomial Q_n. In particular, for the Legendre polynomials, this weighting factor is K_n and the square RMS value is given by:

$$RMS^2 = (1/2)\sum C_n{}^2 K_n$$

Corresponding constructions can be derived for other types of polynomials.

Those of ordinary skill in the art will appreciate that the RMS value of the synthesized signal according to equation (4) is equivalent to the summation of the coefficients C_n on a square law basis followed by a square root calculation.

With an application of the non-restrictive illustrative embodiment of the filtering technique according to present invention to EMG signal processing, it has been observed that the polynomials of lowest orders mimic the baseline variations and

polynomials of somewhat higher orders describe the ECG quite well. It has further been observed that the above-described second and third method automatically eliminate high frequency noise.

It should also be pointed out that Legendre polynomials of higher orders present an oscillatory behaviour thereby making it possible to obtain estimates for the dominating periodicity of the wanted signal components. The weighting coefficients C_n are summed on a square law basis with a weighting proportional to the number of oscillations for the order n of the polynomial Q_n, normalized with respect to the sum of the weighting coefficients C_n on a square law basis with polynomial order weighting K_n, in order to obtain the dominating periodicity of the wanted signal components. This provides an estimated number of oscillations (FRQ) in the signal interval $-1 \leq x \leq 1$:

$$FRQ = \frac{\sum C_n^2 K_n (n/2)}{\sum C_n^2 K_n}$$

Similar constructions can be derived for other types of polynomials.

Windowing to remove edge effects

Equation (4) shows that an infinite number of polynomials are needed to fully represent the characteristics of the signal. If the summation is done over a limited number of polynomials, edge effects similar to those found in Fourier synthesis will occur. These edge effects are constituted by oscillations whose strength and frequency depend on the number of sinusoidal harmonic terms that are summated. More specifically, the edge effects can be described as being constituted by imperfections having a strength depending on the limited number of summated polynomials. The orthogonal polynomials, as described in Table 1 and illustrated in Figure 2, attain large values at the opposite ends of the limited time interval. There is also a shift in sign between polynomials of adjacent orders at x=-1.

These edge effects may have a large influence on the filtered signal and have to

be considered. To suppress the influence of the edge effects on the filtered signal the following edge effect removal method according to a non-restrictive illustrative embodiment of the present invention can be used:

Operation 601 (Figure 6)

Overlapping windows are first defined. As a non-limitative example, 50% overlapping windows such as odd windows 74 and even windows 75 of Figure 7 can be used. Although the example of Figure 7 uses 50% overlapping windows, it is within the scope of the present invention to use many other types of suitable overlapping windows.

Operation 602 (Figure 6)

Filtering in accordance with one of the above-described methods is conducted in the overlapping windows, for example the windows 74 and 75 in Figure 7.

Operation 603 (Figure 6)

A weighting is conducted to suppress the signal at the opposite ends of each window and emphasize the signal in the central part of the same windows.

In the non-restrictive, illustrative embodiment of the filtering technique according to the present invention, the signal weighting is conducted in the various windows by means of respective edge effect suppressing functions constructed in such a way that the sum of the functions from the different overlapping windows is equal to unity (see for example functions 101 and 102 of Figure 10).

In a non-restrictive illustrative embodiment of the present invention, since the sum of the weighting function for windows 74 and the weighting function for windows 75 is equal to unity, and the value of the weighting functions at the opposite ends of the respective windows is equal to zero in order to suppress the edge effects, the following conditions are met for such a weighting window:

$$W(x{=}-1) = W(x{=}+1) = 0 \quad (6)$$

and

$$W(x) + W(1-x) = 1 \quad (7)$$

This implies that

$$W(0) = 1 \quad (8)$$

and

$$W(-x) = W(x) \quad (9)$$

There are a number of functions capable of fulfilling these conditions. Two might be mentioned: a triangular function (equation (10a)) and a squared cosine function (equation (10b)).

$$W(x) = 1 - |x| \quad (10a)$$

$$W(x) = \cos^2(\pi x/2) \quad (10b)$$

The function of equation (10b) presents the characteristics of having continuous derivatives and to suppress the signal more efficiently at the opposite ends of the window.

Operation 604 (Figure 6)

The overlapping signal parts (see for example signal parts 111 and 112 of Figure 11) are then summated to give the output, filtered signal (see for example signal 122 of Figure 12) free from the unwanted signal components.

Example (Figures 7-12)

A non-limitative example, corresponding to a combination of the first filtering method (Figure 3) and windowing (Figure 6), will now be described with reference to Figures 7-12.

Referring to Figure 7, the input contaminatied signal (upper trace 71) is splitted into 50% overlapping windows such as 74 and 75 numbered in sequence order, including odd windows 74 (middle trace 72), and even windows 75 (lower trace 73).

In this non-limitative example, as illustrated in Figure 8, the unwanted signal components in the middle 72 and lower 73 traces of Figure 7 are modeled in every window 74 and 75 as signals 81 and 82, respectively, synthesized from Legendre polynomials of orders zero (0) to seven (7).

Figure 9 shows the signal parts 91 remaining after removal of the signal 81 (Figure 8) in the odd windows 74 (middle trace 72) of Figure 7. In the same manner, Figure 9 shows the signal parts 92 remaining after removal of the signal 82 (Figure 8) in the even windows 75 (lower trace 73) of Figure 7.

Figure 10 illustrates an edge effect suppressing function 101, corresponding to equation (10b), in each odd window 74 for smoothing transitions from one odd window 74 to the other. In the same manner, Figure 10 illustrates an edge effect suppressing function 102, corresponding to equation (10b), in each even window 77 for smoothing transitions from one even window 75 to the other.

In Figure 11, the signal parts 111 of the odd windows are obtained by multiplying the remaining signal parts 91 of Figure 9 by the edge effect suppressing function 101 of the odd windows 74. In the same manner, the signal parts 112 (even windows) of Figure 12 are obtained by multiplying the remaining signal parts 92 of Figure 9 by the edge effect suppressing function 102 of the even windows 75. Multiplication of the remaining signal parts 91 by the function 101 has the effect of suppressing the above-described edge effects related to the odd windows 74. In the same manner, multiplication of the

remaining signal parts 92 by the function 102 has the effect of suppressing the above-described edge effects related to the even windows 75.

Finally, in Figure 12, the upper trace 121 corresponds to the input contaminated signal to be filtered as illustrated in the upper trace 71 of Figure 7. The lower trace 122 of Figure 12 is the processed signal from which the unwanted signal components have been filtered out, and corresponds to the summation of signal parts 111 (corresponding to the odd windows 74) and 112 (corresponding to the even windows 75) of Figure 11.

Although the present invention has been described hereinabove in connection with illustrative embodiments thereof, these illustrative embodiments can be modified at will, within the scope of the appended claims without departing from the spirit and nature of the present invention.